# Every Student Counts

## Professional Development Guide
## High School Level

Year 2 - Day 1

Iowa Department of Education

# Professional Development Plan
## High School - Year 2 - Day 1
## Outline for the Day

**Content Goals:**

NCTM Geometry Standard: Visualization and Modeling

- Use visualization, spatial reasoning, and geometric modeling to solve problems about geometric relationships

Mathematical Themes:

- Use geometric modeling to solve problems
- Use visualization and reasoning to solve problems
- Visualize, construct, and represent 3D shapes

**NCTM Principle**: Equity

**NCTM Process Standard**: Representation

**Overall Teaching Goal**: Teaching and learning mathematics through problem solving

| Activity | Description for Facilitator | Time (Min) | Teacher Masters (TM) & Materials |
|---|---|---|---|
| 1. Welcome and overview | ▪ Welcome & Introductions<br>▪ Review Theory-Demonstration-Practice<br>▪ Review Year 2 Outline<br>▪ Review Day 1 Overview<br>▪ Review Day 1 agenda | 20 | TM-1: Year 2 Outline<br>TM-2: Day 1 Overview<br>TM-3: Day 1 Agenda<br>Powerpoint file: Day1slides.ppt |
| 2. Discussion of Assigned Readings | Round-robin chart-paper activity to summarize and discuss readings | 30 | Chart paper, marking pens<br>TM-4: Day 1 assigned readings<br>TM-5: Past Participant Comments |
| 3. Focus on Teaching and Learning | Briefly present and discuss research and main points for two instructional strategies related to ESC: questioning and collaborative learning. | 20 | TM-6: Collaborative Learning<br>TM-7: Roles for Collaborative Learning<br>Powerpoint file: Day1slides.ppt |

| Activity | Description for Facilitator | Time (Min) | Teacher Masters (TM) & Materials |
|---|---|---|---|
| 4. PBIT 1-Use Geometric Modeling to Solve Problems: Vertex-Edge Graphs | ▪ Engage in a lesson on solving conflict problems using vertex-edge graphs<br>▪ Follow-up discussion and examples of assessment - assessment for learning | 75 | TM-8: Solving Conflict Problems with Vertex-Edge Graphs<br>TM-9: Assessment: Solving Conflict Problems with Graphs (and Solutions)<br>TM-10: Sample formative assessment checklist<br>TM-11: Overhead masters |
| 5. Debrief and Review ESC | ▪ Use the previous activity as a springboard to review two of the major components of ESC: PBIT and MDP<br>▪ Discuss and review the new assessment component of ESC: Assessment for Learning | 30 | Powerpoint file: Day1slides.ppt |
| 6. PBIT 2 - Visualize, Construct, and Represent 3D Shapes: Cube Structures | ▪ Engage in a lesson on building, visualizing, and drawing cube structures<br>▪ Follow-up discussion on assessment, questioning, and mathematics | 55 | TM-12: Visualizing and Sketching Three-Dimensional Shapes<br>Models of boxes and a “double square pyramid”<br>Powerpoint file: Day1slides.ppt |
| 7. PBIT 3 - Use Visualization and Reasoning to Solve Problems: Platonic Solids | ▪ Engage in a lesson to answer the question: How many regular polyhedra (Platonic Solids) are there?<br>▪ Follow-up discussion on assessment and mathematics | 55 | TM-13: How Many Regular Polyhedra (Platonic Solids) Are There?<br>TM 14: Regular polygon template<br>TM 15-19: Cutout templates (nets) for the regular polyhedra<br>Models of the regular polyhedra<br>Scissors, tape, glue sticks, rulers<br>Access to the Internet |
| 8. Closure | ▪ Summarize and provide closure for the day, referring to the NCTM content standard, process standard, and principle for the day<br>▪ Give Day 2 assignment<br>▪ Workshop evaluation | 15 | TM-20: Reading Assignment for Day 2<br>Agenda for Day 1 with assignment at the bottom<br>Workshop evaluation forms<br>Powerpoint file: Day1slides.ppt |

## Activity 1: Welcome and Overview

Time: 20 minutes

**Overview and Rationale:**
Begin the day with introductions and an overview of the year and the day. It is important that everyone sees and understands the overview before getting into all the details of the ensuing activities.

**Conducting the Activity:**
Welcome everyone and make necessary introductions.
Present an overview of the year and the day by presenting and briefly dicussing the following three handouts:

1. Theory-Demonstration-Practice in Year 2
   - Briefly discuss the 1st slide in the file Day1slides.ppt

2. Year 2 Outline Chart
   - Review the big picture for the whole year, and where Day 1 fits in.

3. Day 1 Overview Chart
   - Remind participants of the 3 main themes of Every Student Counts: (i) teaching for understanding, (ii) problem-based instructional tasks, and (iii) meaningful distributed practice.
   - Point out that these 3 themes will be applied to the focal points for Day 1: geometric visualization and modeling (mathematical content), equity (NCTM principle), and representation (NCTM process standard).
   - Point out the main Day 1 activities at the bottom of the page.

4. Day 1 Agenda
   - Briefly go through the agenda for the day.
   - Point out the assignment on the bottom of the page but discuss this later, at the end of the day.

**Materials**
- 3 handouts listed above (Teaching Masters TM 1-3)
- Powerpoint file: Day1slides.ppt

TM-1

# Every Student Counts – High School
# Year 2 Outline 2005-2006

**Content Focus: Geometry**
**Teaching Focus: Teaching and Learning Mathematics through Problem Solving**

| | **Day 1**<br>**October 25/26** | **Day 2**<br>**December 6/7** | **Day 3**<br>**February 14/15** | **Day 4**<br>**April 18/19** |
|---|---|---|---|---|
| **NCTM Content Standard** | Visualization and Modeling | Coordinate Geometry | Transformations | Properties and Proof |
| | Use visualization, spatial reasoning, and geometric modeling to solve problems | Specify locations and describe spatial relationships using coordinate geometry and other representational systems | Apply transformations and use symmetry to analyze mathematical situations | Develop mathematical arguments about geometric relationships |
| **Mathematical Themes** | • Use geometric modeling to solve problems<br>• Use visualization and reasoning to solve problems<br>• Visualize, construct, and represent 3D shapes | • Different coordinate systems<br>• Coordinates reshape and connect algebra and geometry<br>• Coordinates and technology | • Symmetry<br>• Transformations<br>• Computer graphics | • Different types of proof<br>• Different types of geometry<br>• Inductive reasoning --> deductive proof |
| **Mathematical Activities** | • Vertex-edge graphs<br>• Representations of block structures<br>• Platonic solids | • Rectangular vs. angular coordinate systems<br>• Software and coordinate investigation of geometric properties<br>• Contour maps | • What is symmetry<br>• Strip symmetries<br>• Reflect This<br>• Animation | • Same property, different proofs<br>• How many parallel lines?<br>• Data, conjecture, proof |
| **NCTM Principle** | Equity | Technology | Teaching | Learning |
| **NCTM Process Standard** | Representation | Connections | Communication | Reasoning and Proof |
| **Assessment** | Assessment is embedded in problem-based teaching | Questioning for assessment | Assessment for Learning | Assessment for Learning |
| **Technology/ Manipulative Tools** | • Blocks<br>• Construction paper | • Dynamic geometry software<br>• Graphing calculators<br>• Applets<br>• Simulated topographical terrain | • Real-world objects and images<br>• Reflection tool<br>• Applets | • Dynamic geometry software<br>• Spheres<br>• Applets |

TM-2

# Every Student Counts – High School Day 1

## Overview

• Teach for Understanding
• Focus on Meaning

• Problem-Based Instructional Tasks
• Teaching through Problem Solving

Meaningful Distributed Practice
of Concepts, Skills, and Problem Solving

**Goals for Today**
• Visualization and Modeling
• Representation
•Equity

**Activities for Today**
• Use Visualization and Reasoning to Solve Problems: Platonic Solids
• Use Geometric Modeling to Solve Problems: Vertex-Edge Graphs
• Visualize, Construct, and Draw 3D Shapes: Cube Structures
• Debrief and Review ESC

# TM-3 Every Student Counts – High School Year 2-Day 1

## Agenda

NCTM Geometry Standard: Visualization and Modeling
NCTM Principle: Equity
NCTM Process Standard: Representation

1. Welcome, Introductions, Overview of Year and Day

2. Readings Discussion

3. Focus on Teaching and Learning

4. Problem-Based Instructional Task 1: Use Geometric Modeling to Solve Problems
   Solve Conflict Problems with Vertex-Edge Graphs

5. Debrief and Review ESC
   PBIT, MDP, Lesson Format, Assessment for Learning

6. Problem-Based Instructional Task 2: Visualize, Construct, and Represent 3D Shapes
   Build, Visualize, and Draw Cube Structures

7. Problem-Based Instructional Task 3: Use Visualization and Reasoning to Solve Problems
   How Many Regular Polyhedra (Platonic Solids) Are There?

8. Closure, Assignments, Evaluations

**Assignments (Due on Day 2)**

1. Reading assignment – see attached
2. Classroom Practice Assignment

   There are two general classroom practice assignments: enriching lessons and assessment.
   - Enriching Lessons Assignment 1 – Find a cooperating teacher who is teaching geometry. Make contact, agree to enrich a lesson in the spring.
   - Assessment for Learning Assignment 1a – Observe a class taught by your geometry cooperating teacher. Briefly describe at least 1 example of effective questioning to assess learning (see Reading assignment). Also, briefly describe at least 1 example of questioning that could be improved. Bring these descriptions to Day 2, and be prepared to present and discuss them in small groups.
   - Assessment for Learning Assignment 1b – Revisit or find a new algebra cooperating teacher, for one last algebra activity (which also relates to coordinate geometry). Administer the assessment on “Some Problems About Graphs” (see Reading assignment). Collect and examine the student results. Bring the results and be prepared to present and discuss them in small groups on Day 2.

## Activity 2: Discussion of Assigned Readings

Time: 30 minutes

**Overview and Rationale**

This small group activity gives participants the opportunity to identify and briefly discuss the main points in the assigned readings. Doing this early in the day gets the main points from the readings out on the table so that they can be applied and connected to the day's activities.

**Connections to Other Activities and the Whole Day**

The assigned readings are chosen so that they are relevant to the day's activities. Thus, discussion of the readings comes early in the day so that we can connect, clarify, and apply the main points in the readings throughout the day.

**Conducting the Activity**

- **Grouping**
  Form a number of groups equal to the number of readings (or a multiple). Form the groups so that at least one person with secondary mathematics teaching expertise is in each group.
- **Discussion Method**
  Each group is initially assigned one of the readings. They generate main points, important questions, and/or key examples from that reading and write these on a sheet of chart paper. After 5 minutes, the sheets are passed on to an adjacent group. Each group reads the points that the previous group listed for that reading and then adds more points, questions, or examples. This process continues until each group has considered each reading. All charts are taped to the wall for viewing during the rest of the day and for reference during the remaining activities of the day.
- **Time**
  Allow 5 minutes to transition to and from the activity, and 5 minutes for each group to discuss each reading. Total time: 30 minutes.

**Materials**

- Chart paper and marking pens for each group
- Tape (or self-adhesive chart paper)
- Copy of assigned readings for Day 1, for reference (TM-4)
- Copy of responses from previous participants, for reference (TM-5)

TM-4

# Every Student Counts – High School Year 2
# Reading Assignment for Day 1

**Assessment**

In *Every Student Counts* we focus on problem-based instructional tasks, meaningful distributed practice, and teaching for understanding. This year we are adding a new dimension – assessment. The following readings consider the role of assessment (and questioning) in a problem-based classroom.

1. Read Chapter 11, "The Sound of Problem Solving" by Driscoll in *Teaching Mathematics through Problem Solving Grades 6 – 12*, pages 161-175.
   - This reading provides guidelines for effective listening to students as they work alone and in groups on problems with the purpose of better understanding what and how the students are thinking as they learn important mathematical ideas.
   - Reflection Question: Compare and contrast the five main question types in the "taxonomy of questioning intentions" on pages 173-4. What are some other questions of each type?
2. Read Chapter 12, "Classroom Assessment Issues Related to Teaching Mathematics through Problem Solving" by Ziebarth in *Teaching Mathematics through Problem Solving Grades 6 – 12,* pages 177-189.
   - This chapter describes some of the characteristics of good assessment in problem-based classrooms and discusses some challenges that teachers may find in adopting such assessment.
   - Reflection Question: The author raises four practical issues for teachers related to assessment in a problem-based classroom (see pp. 180-81). Which of these (or others that you identify) do you think will require the most difficult adjustment for high school mathematics teachers in your area?

**NCTM Principles**

NCTM's *Principles and Standards for School Mathematics* presents six Principles that describe essential features of high quality mathematics education: Equity, Technology, Teaching, Learning, Curriculum, and Assessment. The Assessment Principle is a theme that will run throughout Year 2 of Every Student Counts. The Curriculum Principle will be a major focus in Year 3. The other four Principles will each be the focus of one of the days in Year 2. Equity is the Principle for Day 1.

3. Read the section on the Equity Principle in Chapter 2: Principles for School Mathematics from *Principles and Standards for School Mathematics*.
   - The Equity Principle states that: "Excellence in mathematics education requires equity–high expectations and strong support for all students."
   - Reflection Question: There are three main points made in the description of the Equity Principle. How is each of these key points about equity reflected, or not, in mathematics classrooms in your area?

**NCTM Process Standards**

NCTM's *Principles and Standards for School Mathematics* presents five content standards and five process standards. The Process Standards are: Representation, Connections, Communication, Reasoning and Proof, and Problem Solving. All the Process Standards are woven throughout *Every Student Counts*. The Problem Solving Standard is implicit in the theme of Problem-Based Instructional Tasks. Each of the other four Standards will be the focus of one of the days of Year 2. Day 1 focuses on Representation.

4. Read "Representation" for Grades Pre-K – 12 (pp. 67-71) and for Grades 9 –12 (pp. 360-364) in *Principles and Standards for School Mathematics*.
   - The Representation Standards consists of three main goals. Be prepared to analyze the Day 1 activities in terms of the three goals of the Representation Standard.
   - Reflection Question: According to these readings, what is the teacher's role in identifying and using multiple representations in mathematics classes?

# TM-5 Past Participant Comments on Assigned Readings:

CLASSROOM ASSESSMENT ISSUES RELATED TO TEACHING MATHEMATICS THROUGH PROBLEM SOLVING
By Ziebarth
*Teaching Mathematics through Problem Solving Grades 6-12, pages 177-189*

- How do you do it? (with quiet or unmotivated students)
- How do you get teachers to change assessment practices?
- Where is the line of best fit between assessing content and process? 50% quizzes and exams?
- Rubrics: construction and time to construct
- Not only do teachers need to expand thinking about kinds of assessment, but so do students (and parents)
- Listening to/fostering mathematical discussion
- Align assessment to match instruction
- Students should be aware of assessment criteria (open ended assessment is difficult to grade)
- Rubrics are helpful
- Assessment should be used to help design instruction
- Group vs. individual assessment (finding a balance)
- Requires time for teachers to become comfortable with problem-solving type assessment
- Adopting new grading criteria is of high concern for teachers
- Important to have student input
- Parent acceptance
- 2 goals: formative – monitor student math understanding and summative – to evaluate and assign a grade
- Multiple sources
    - carefully chosen questions
    - group observations
    - whole class conversations
    - journals
    - in class/take home assignments
    - portfolios
    - projects
- Requires teacher and student metacognition
- Teacher needs to get to student thinking
- Assigned letter grade - what does an "A" mean?
- Observation
- Change assessment format
- Student level-help students to understand new ways of assessment
- Teachers need to have multitude of assessment
- Allow time for teachers to be trained and implement
- How to use formative assessment to improve learning
- The problem isn't as important as the questions you ask
- Teachers have issues with grading group work
- Students should be aware of the assessment standards
- What will be done with assessment data?
- Kids should know what rubric criteria is

- Communicate to parents/students/administration…what are the purposes/formats for assessment?
- Equity in assessments
- Two reasons for assessment:
  - monitor for understanding so teachers have data to drive future instruction
  - assign a grade
- Check for skill development then for deep understanding
- Assessments from group work a HOT TOPIC – how do you assign individual grades?
- Assess what you teach and teach what you assess
- Adjustment for change in style for teachers, students, and families
- Not quick change – make it a process
- Assessments should be grade-level specific
- P. 187 assessment that demonstrates the highest achievement in students – teachers' use of methods designed to assess understanding has been found to be associated with students' increased learning
- Wide variety of assessment
- All assessments don't look alike for all levels in the classroom

NCTM PROCESS STANDARD – REPRESENTATION
Pre-K-12 pp. 67-71 and grades 9-12 pp. 360-364
*Principles and Standards for School Mathematics*

- Organize, record, and communicate
- Diagrams, graphical displays and symbolic expressions
- Refers to both process and product
- Utilizing multiple representations can provoke interesting classroom discussions which can help the teacher "move the class toward a more nearly accurate representation"
- Teachers can gain insight into students' thinking
- It's an essential component of mathematics
- Multiple representation allows for greater understanding
- Representation applies to external and internal thought processes
- Representations are not the end – they are a bridge for student understanding of math concepts and relationships
- Multiple Representations allow you to highlight – convey different information
- Applying math to realistic situations through modeling
- Safe environment – variety of thinking styles
- Support students in use of/critique of the representations used
- Highlight ways in which different representations of the same objects' processes can convey different information and help kids make connections
- Help students organize and record thinking and mathematical ideas more concrete, complete and available for reflection
- Becomes more abstract and/or complex as students move through H.S.
- Not only can different representations model the same concept or problem, but different problems can be modeled with the same representation
- Representations accurately represent the math
- Making real-world connections to symbolic/conventional methods
- …representation is fundamental to how people understand ideas
- Multiple forms should be selected, applied and translated to solve problems

- Connections with prior knowledge and among mathematical concepts
- Apply to real-world connections
- High school students should be fluent/flexible between representations. Access to multiple representations and able to apply to more complex situations
- Organize, access, solve
- Could be graphical, algebraic, tables/charts, written symbolic
- Great use of technology to demonstrate representation
- Important concept for diverse learners
- To model
- Helps teachers assess students' deeper understanding of mathematical concepts
- Student generated representations provide a window to student thinking
- Connect personal images to more conventional representations
- Use of technology to extend understanding and link to real life contexts
- Promotes flexible thinking
- Fosters equity – students can experience success in a variety of ways

SOUND OF PROBLEM SOLVING
By Driscoll in *Teaching Mathematics through Problem Solving*
*Grades 6-12 pages 161-175*

- Listening – passive vs. active to determine student thinking
- Focused listening – what is listened for, (purpose)
- Student work should include what we hear as well as what we see
- Allow students to work together for an extended period of time without teacher interruption
- Have teachers have more of an awareness to the intent/purpose of the questions they ask students ( not to just ask questions for the sake of asking)
- Include reflective writing
- Effective problem-solving makes use of heuristics, control, beliefs and resources
- What students say does not always match what they write
- Types of questioning shape students responses
- Trigger students thinking; we uncover students thinking by the questions we ask
- Listening needs a focus
- Teachers' assumptions affect what we hear
- Questioning is a skill we can learn
- Higher level questioning
- Be prepared – focus on where to go
- Teachers really need to listen to students
- Students need to know types as well so they can use them
- Teacher listens to student responses
  - -responses to problem solving tasks
  - -answers to questions
  - -problem solving endeavors
- Listen to understand student thinking, not to teach
- Reflective writing to understand one's own thinking
- Willingness to let students struggle
- Classroom climate needs to promote communication expectations and encourage risk taking
- The objective a teacher has in mind effects what they hear

- Teachers need to learn (practice) how to listen with a focus
- Use reflective listening techniques
- Students need the processing time ("We" get impatient when so much time seems to be going by and we interrupt or cut off the time students need.)
- Need to encourage student interaction – communication they can often help another student understand better than we can
- Teachers need to learn to listen to students:
  - -actively or passively
  - -listen to understand students' thinking
  - -listen to assess understanding
- Factors that influence listening:
  - -teachers' beliefs
  - -teachers' experience
  - -teachers' intentions
- Five types of questions
  - -managing
  - -clarifying
  - -orienting
  - -promoting mathematical reflection
  - -eliciting algebraic thinking
- Evaluative listening
- How do you listen to all groups?

NCTM PRINCIPLE – EQUITY
In Chapter 2: Principles for School Mathematics
*Principles and Standards for School Mathematics*

- High expectations (H.S. reform)
- Access to challenging curriculum and competent teacher in classroom and supplemental instruction (resource teacher)
- Equity – not always equal – move <u>all</u> learners forward
- A process rather than a product
- Requires teachers to understand and confront own beliefs and biases (teachers, administrators)
- Accommodations and learning strategies for <u>all</u> students; resources and support
- Technology a resource, can assist with achievement
- Equity does not mean identical
- All students have access to high quality curriculum
- Need to accommodate for differences
- High expectations alone are not sufficient to accomplish equity – instead you need a solid math program
- Use reasonable and appropriate accommodations
- Mathematics can and must be learned by all students
- Professional development for teachers important
- Achieving equity requires a significant allocation of human and material resources. (Time)
- People need to learn <u>how</u> to do accommodations
- Oral rather than written for some students
- Equity requires obligation

- Meeting needs of all types of learners with a variety of techniques – not necessarily identical instruction (multiple representation, technology,….individual accommodations)
- High expectations and worthwhile experiences for all students. Requires more resources than we provide for it
- …without inhibiting the learning of others…
- How to adapt all modalities with time constraints
- Access to high quality curriculum for all students
- PBIT's help
- Same standard (high expectations)
- More time for others (accommodating differences)
- Highly qualified staff
- Teacher behavior
- Acceptance of a variety of learning styles
- Use of assessment to guide instruction
- Varied resources i.e. technology, manipulatives, extra help (people), collaborations
- Accommodations need to be made at both ends of spectrum
- Differentiated instruction and universal design (aligning curriculum and materials to be accessible to all)
- Need increased accessibility of material in other languages
- Teachers need to believe all students can achieve
- High expectations with support

## Activity 3: Focus on Teaching and Learning

Time: 20 minutes

### Overview and Rationale

In this activity we briefly discuss, and present research for, two common instructional strategies that are often used in ESC: questioning and collaborative learning. These strategies are not official components of Every Student Counts, but they are often used in ESC activities and part of a problem-based approach to teaching and learning mathematics.

### Connections

Questioning and collaborative learning have been used often in Every Student Counts, including in the activities today. This activity will present a brief discussion and research base for these strategies.

### Conducting the Activity

- Briefly present and discuss the slides on questioning and cooperative learning in the Day1slides.ppt file.
- Briefly present and discuss the Teaching Masters on Collaborative Learning and Roles for Collaborative Learning (TM 6-7). Point out that these roles will be used today, beginning with the next activity.

### Materials

- TM-6: Collaborative Learning
- TM-7: Roles for Collaborative Learning
- Powerpoint slides from: Day1slides.ppt

TM-6

# Collaboration

- Each member contributes to the group's work.
- Each member of the group is responsible for listening carefully when another group member is talking.
- Each member of the group has the responsibility and the right to ask questions.
- Each group member should help others in the group when asked.
- Each member of the group should be considerate and encouraging.
- Work together until everyone in the group understands and can explain the group's results.

# SOME COLLABORATIVE LEARNING ROLES

## READER / MEASUREMENT SPECIALIST

- Reads out loud and explains the questions or problems on which the group will be working.
- Performs the actual measurements as needed.

## RECORDER

- Writes a summary, using complete sentences, of the group's decisions and ideas, and reads them back to the group to ensure agreement and accuracy.
- Shares group's summary with other groups or the entire class.

## QUALITY CONTROLLER

- Monitors the group's results and makes sure that the group produces high quality work of which they can be proud.
- After each response, checks to see that each group member has recorded the correct response.
- Makes sure the group follows the ground rules.

## COORDINATOR

- Keeps the group on task and makes sure everyone is participating.
- Communicates with the teacher and other coordinators on behalf of the group.
- Obtains the necessary resources.
- Recommends data gathering methods and units of measure appropriate for the situation.

# Activity 4
# Problem-Based Instructional Task 1
# Use Geometric Modeling to Solve Problems:
# Solve Conflict Problems with Vertex-Edge Graphs

Time: 75 minutes

**Overview and Rationale**

This activity is the first problem-based instructional task for today. It illustrates the use of geometric modeling to solve problems, in particular the use of vertex-edge graph models. Vertex-edge graphs are widely used in business and industry and recommended for all students in NCTM's *Principles and Standards for School Mathematics*.

**Connections**

This activity will illustrate and apply some of the collaborative learning points discussed in Activity 3. It illustrates the ESC lesson format and also several points about assessment. It will also serve as the stimulus for the next activity: Debrief and Review ESC.

**Conducting the Activity**

Participants should work in small collaborative groups to complete the lesson on Solving Conflict Problems with Vertex-Edge Graphs. (See Teaching Master TM-8.) This should be carried out as a classroom simulation, with the workshop instructor acting as the teacher and the participants engaged as students. In particular:

- The lesson is organized in the ESC lesson format: Launch-Explore-Summarize-Check For Understanding. Be sure to model all phases of this lesson format.
- As you circulate, be sure to ask good questions, reinforce good group behavior, and complete a sample formative assessment checklist (see below and TM-10).
- Point out the focus questions at the beginning of the lesson, and discuss how they are reflected in the Summarize and the Check for Understanding.
- An assessment/quiz is also included. Use this as time permits. You could have those groups that finish early do this quiz. For everyone, you should at least present the assessment/quiz and discuss why and how if we teach with problem-based instructional tasks then we must also assess with problem-based instructional tasks.
- Follow-up discussion on assessment: Point out the role and examples of assessment in this activity – Summarize questions, Check for Understanding, questioning during the Explore and Launch, the sample assessment/quiz, the sample formative assessment checklist.

**Materials**

- TM-8: Solving Conflict Problems with Vertex-Edge Graphs
- TM-9: Assessment: Solving Conflict Problems with Graphs (and Solutions)
- TM-10: Sample formative assessment checklist
- TM-11: Overhead masters (4 pages)

# Solving Conflict Problems with Vertex-Edge Graphs

n the last lesson, you learned about vertex-edge graphs—geometric diagrams in which shape is not important, only the relationship between vertices as defined by the edges is essential. You used these graphs to model and solve problems related to paths and circuits. They can also be used to solve many other types of problems. In this lesson, you will investigate how graphs can be used to avoid possible conflict among a finite number of objects, people, or other things.

To begin, consider the problem of assigning radio frequencies to stations serving the same region. In cities and towns, you can listen to many different radio stations. Each radio station has its own transmitter which broadcasts on a particular channel, or frequency. The Federal Communications Commission (FCC) assigns the frequencies to the radio stations. The frequencies are assigned so that no two stations interfere with each other. Otherwise, you might tune into "Rock 101.7" and get Mozart instead!

## Launch

Suppose seven new radio stations have applied for permits to start broadcasting in the same region of the country.

**a** What are some factors that may need to be considered by the FCC as they decide how to assign frequencies to these stations?

**b** How does this situation involve "conflict"?

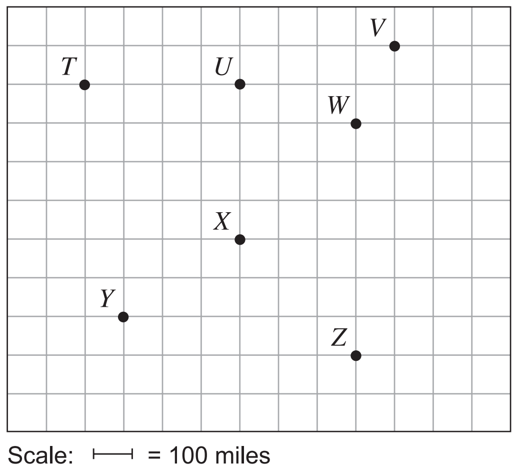
**c** How do you think the FCC should assign frequencies to the seven stations?

**d** Why do you think the FCC might like to assign the fewest possible number of new frequencies for the seven stations?

In this lesson, you will learn how to use vertex-edge graphs to solve problems such as assigning non-interfering radio frequencies, using a technique called *vertex coloring*.

## Explore

Suppose the seven new radio stations that have applied for broadcast permits are located as shown on the grid at the right. *A side of each small square on the grid represents 100 miles.* The FCC wants to assign a frequency to each station so that no two stations interfere with each other. The FCC also wants to assign the fewest possible number of new frequencies. Suppose that because of geographic conditions and the strength of each station's transmitter, the FCC determines that stations within 500 miles of each other must be assigned different frequencies.




*Your work on the problems of this investigation will help you answer this question:*

How can vertex-edge graphs be used to assign frequencies to these seven radio stations so that as few frequencies as possible are used and none of the stations interfere with each other?

**1.** For a small problem like this, you could solve it by trial and error. However, a more systematic method is needed for more complicated situations. Working on your own, begin modeling this problem with a graph. Remember, *to model a problem with a graph, you must first decide what the vertices and edges represent.*

- **a.** What should the vertices represent?
- **b.** How will you decide whether or not to connect two vertices with an edge? Complete this statement:

  *Two vertices are connected by an edge if . . . .*

- **c.** Now that you have specified the vertices and edges, draw a graph for this problem.

**2.** Compare your graph with others in your group.

- **a.** Did everyone in your group define the vertices and edges in the same way? Discuss any differences.
- **b.** For a given situation, suppose two people define the vertices and edges in two different ways. Is it possible that both ways accurately represent the situation? Explain your reasoning.

**c.** For a given situation, suppose two people define the vertices and edges in the same way. Is it possible that their graphs have different shapes but both are correct? Explain your reasoning.

**3.** A common choice for the vertices is to let them represent the radio stations. Edges might be thought of in two ways, as described in Parts a and b below.

**a.** You might connect two vertices by an edge whenever the stations they represent are 500 miles or *less* apart. Did anyone in your group do this? If not, draw a graph where two vertices are connected by an edge whenever the stations they represent are 500 miles or *less* apart.

**b.** You might connect two vertices by an edge whenever the stations they represent are *more* than 500 miles apart. Did anyone in your group do this? If not, draw a graph where two vertices are connected by an edge whenever the stations they represent are *more* than 500 miles apart.

**c.** Compare the graphs from Parts a and b.

**i.** Are both graphs accurate ways of representing the situation?

**ii.** Which graph do you think will be more useful and easier to use as a mathematical model for this situation? Why?

**4.** For the rest of this investigation, use the graph where edges connect vertices that are 500 miles or less apart. Make sure you have a neat copy of this graph.

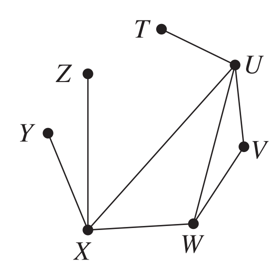
**a.** Are vertices (stations) *X* and *W* connected by an edge? Are they 500 miles or less apart? Will their broadcasts interfere with each other?

**b.** Are vertices (stations) *Y* and *Z* connected by an edge? Will their broadcasts interfere with each other?

**c.** Compare your graph to the graph at the right.

**i.** Explain why this graph also accurately represents the radio-station problem.

**ii.** What criteria can you use to decide if two graphs both represent the same situation?




**5.** Remember that the problem is to assign frequencies so that there will be no interference between radio stations. So far, your graph models this problem as follows. Vertices represent the radio stations. Two vertices are connected by an edge if the corresponding radio stations are within 500 miles of each other. Here's the last step in building the graph model—represent the frequencies as *colors*. So now, assigning frequencies to radio stations means to assign colors to the vertices.

Examine the statements in the following partially-completed table. Translate each statement about stations and frequencies into a statement about vertices and colors. (The first one is already done for you.)

| Statements about stations and frequencies | Statements about vertices and colors |
|---|---|
| Two stations have different frequencies. | Two vertices have different colors. |
| Find a way to assign frequencies so that stations within 500 miles of each other get different frequencies. | |
| Use the fewest number of frequencies. | |

**6.** Now use as few colors as possible to **color the graph** for the radio-station problem. That is, assign a color to each vertex so that any two vertices that are connected by an edge have different colors. You can use colored pencils or just the names of some colors to do the coloring. Color or write a color code next to each vertex. Try to use the smallest number of colors possible.

**7.** Compare your coloring with that of another group.

- **a.** Do both colorings satisfy the condition that vertices connected by an edge must have different colors?
- **b.** Do both colorings use the same number of colors to color the vertices of the graph? Reach agreement about the minimum number of colors needed.
- **c.** Explain, in writing, why the graph cannot be colored with fewer colors.
- **d.** For two particular vertices, suppose one student colors both vertices red while another student colors one vertex red and the other blue. Is it possible that both colorings are acceptable? Explain your reasoning.
- **e.** Describe the connection between graph coloring and assigning frequencies to radio stations.

## Summarize

Some problems can be solved by coloring the vertices of an appropriate graph.

- **a** What do the vertices, edges, and colors represent in the graph that you used to solve the radio-station problem?
- **b** How did “coloring a graph” help solve the radio-station problem?
- **c** Describe some strategies or algorithms that you used to color the graph.
- **d** In what ways can two graphs differ and yet still both accurately represent a situation?

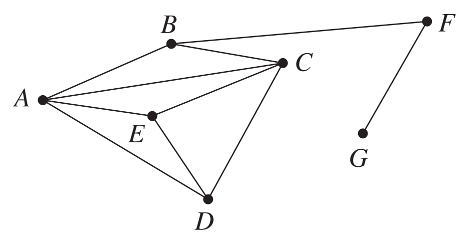
***Be prepared to share your thinking and coloring algorithm with the class.***

Finding good graph-coloring algorithms is an active area of mathematical research with many applications. It has proven quite difficult to find an algorithm that colors the vertices of any graph using as few colors as possible. You often can figure out how to do this for a given small graph, as you have done in this investigation. However, no one knows an efficient algorithm that will color *any* graph with the *fewest* number of colors. This is a famous problem in mathematics, which at the time this activity was written was still unsolved.

## Check for Understanding

1.

Color the vertices of the graph at the right. (As always, this means to use as few colors as possible.)




2.

Hospitals must have comprehensive and up-to-date evacuation plans in case of an emergency. A combination of buses and ambulances can be used to evacuate most patients. Of particular concern are patients under quarantine in the contagious disease wards. These patients cannot ride in buses with non-quarantine patients. However, some quarantine patients can be transported together. The records of who can be bused together and who cannot are updated daily.
Suppose that on a given day there are six patients in the contagious disease wards. The patients are identified by letters. Here is the list of who cannot ride with whom:

| | |
|---|---|
| *A* cannot ride with *B*, *C*, or *D*. | *D* cannot ride with *A* or *C*. |
| *B* cannot ride with *A*, *C*, or *E*. | *E* cannot ride with *F* or *B*. |
| *C* cannot ride with *A*, *B*, or *D*. | *F* cannot ride with *E*. |

The problem is to determine how many vehicles are needed to evacuate these six patients. Use graph coloring to solve this problem. Describe the conflict and state what the vertices, edges, and colors represent.

# Assessment: Solving Conflict Problems with Graphs

Name: ________________________________
Date: _________________________________

**1.** The Olympia Kiwanis Club has the following membership on committees.

Finance—Louganis, Yamaguchi, Joyner-Kersee, Galindo

Publicity—Yamaguchi, Joyner-Kersee, Van Dyken

Membership—Washington, Adams

Social—Washington, Yamaguchi, Martinez

Community Involvement—Adams, Blair, Shuwei

No person can be in two meetings at once. Use graph coloring to find the least number of meeting times needed to schedule a meeting for each committee.

**a.** Draw a vertex-edge graph that represents this situation. Explain the meaning of the edges and vertices in your graph.

*Graph*

*Meaning of vertices:*

*Meaning of edges:*

**b.** Find the least number of meeting times needed. Explain how you can be sure this is the least number from your graph coloring.

*Least number of meeting times:* ______

*Explanation:*

**2.** Ounces, a chemical manufacturing company, wants to build a number of storage buildings for the six hazardous chemicals it produces. The company could build a separate storage building for each chemical, but that would be very costly. Instead, they store some chemicals in the same building, following the guidelines of the federal safety regulations summarized in the table below. Here the chemicals are labeled *A* through *F*, and a "No" indicates that the pair of chemicals in that row and column *cannot* be stored together.

| | ***A*** | ***B*** | ***C*** | ***D*** | ***E*** | ***F*** |
|---|---|---|---|---|---|---|
| ***A*** | | | No | No | | No |
| ***B*** | | | No | No | No | No |
| ***C*** | No | No | | | | No |
| ***D*** | No | No | | | No | No |
| ***E*** | | No | | No | | No |
| ***F*** | No | No | No | No | No | |

**a.** Draw a graph that models this situation. What do the edges and vertices represent in your graph?

*Graph*

*Meaning of vertices:*

*Meaning of edges:*

**b.** Use your graph to determine the least number of storage buildings the plant must build. Explain how you arrived at your answer and why it is the minimum number required.

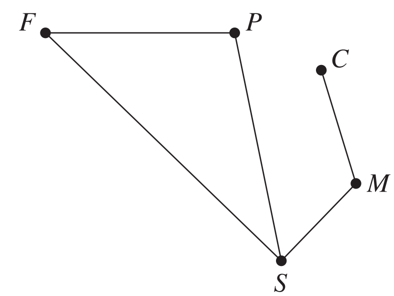
*Least number of storage buildings:* ______

*Explanation:*

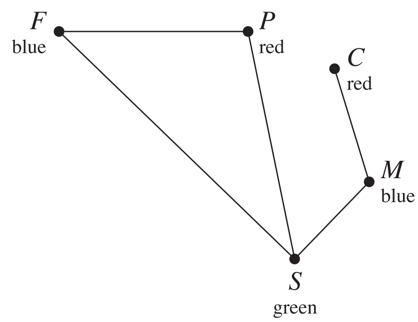
## Assessment:
## Solving Conflict Problems with Graphs

### *Suggested Solutions*

**1. a.** See the graph at the right. Each vertex represents a committee. An edge connects two vertices if the two committees have a member in common.

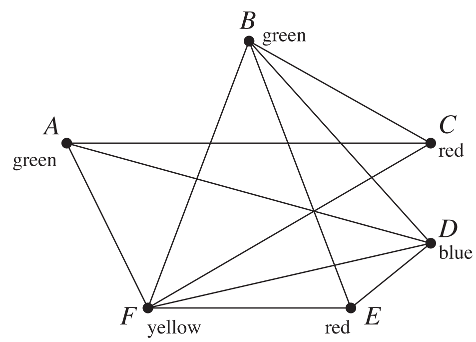



**b.** The least number of meeting times is 3. The colors represent meeting times. Thus, coloring the graph so that two vertices connected by an edge have different colors corresponds to assigning meeting times so that two committees that share a member will have different meeting times. *F*, *P*, and *S* must have different colors because they are all adjacent to each other, so there must be three meeting times.




**2. a.** Let vertices represent chemicals and edges represent a conflict with storing them together. One possible graph and coloring is shown at the right.

**b.** The least number of storage buildings corresponds to the fewest different colors needed to color the graph. The coloring at the right uses four colors. This is the minimum number of colors needed (notice vertices *B*, *D*, *E*, and *F* form a complete subgraph). Therefore, the company must build at least four buildings.



# Sample Formative Assessment Checklist

ESC-Cedar Rapids (E) 2005-2006 (Click for table in MS Excel™ file format)

| Name | | Color a graph | Use coloring to solve a problem |
|---|---|---|---|
| *Sue | | | |
| Art | | | |
| Chuck | | | |
| Deb | | | |
| Gary | | | |
| Jack | | | |
| Mark | | | |
| Mike | | | |
| Vicki | | | |
| *Shelley | | | |
| Lisa | | | |
| Sharma | | | |
| *Bonnie | | | |
| Clair | | | |
| Ron | | | |
| *Terry | | | |
| Brian | | | |
| Jeff | | | |
| Pat | | | |
| Nancy | | | |
| Roger | | | |
| Sally | | | |
| *Laura | | | |
| Alissa | | | |
| Hildasue | | | |
| Larry | | | |
| *Chris | | | |
| Connie | | | |
| Julia | | | |
| Karen | | | |

# Overhead Masters

## Launch

Suppose seven new radio stations have applied for permits to start broadcasting in the same region of the country.

a. What are some factors that may need to be considered by the FCC as they decide how to assign frequencies to these stations?

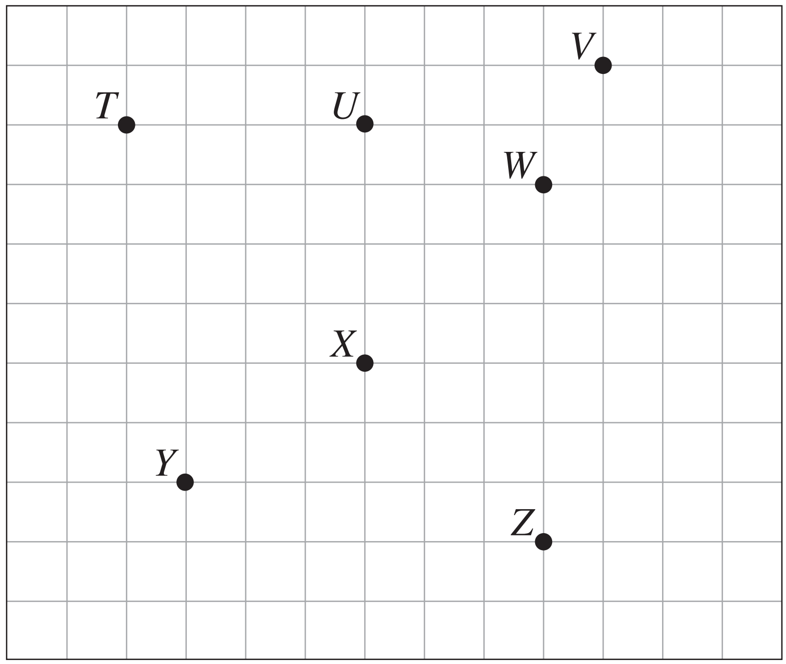
b. How does this situation involve "conflict"?

c. How do you think the FCC should assign frequencies to the seven stations?

d. Why do you think the FCC might like to assign the fewest possible number of new frequencies for the seven stations?

# Radio-Station Problem

**Problem 1**




Scale: ⊢—⊣ = 100 miles

# Modeling the Radio-Station Problem

**Problem 5**

| **Statements about stations and frequencies** | **Statements about vertices and colors** |
|---|---|
| Two stations have different frequencies. | Two vertices have different colors. |
| Find a way to assign frequencies so that stations within 500 miles of each other get different frequencies. | |
| Use the fewest number of frequencies. | |

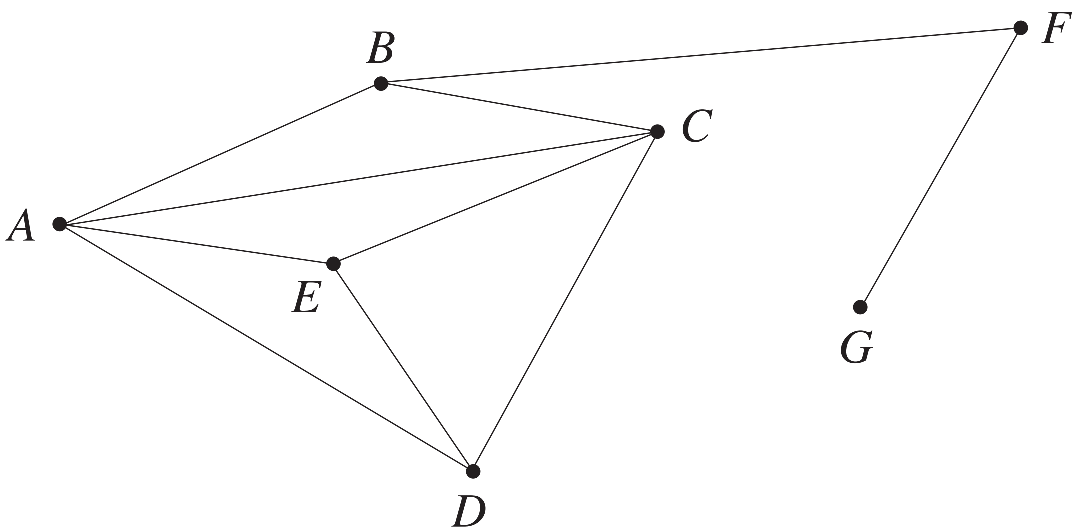
## Summarize

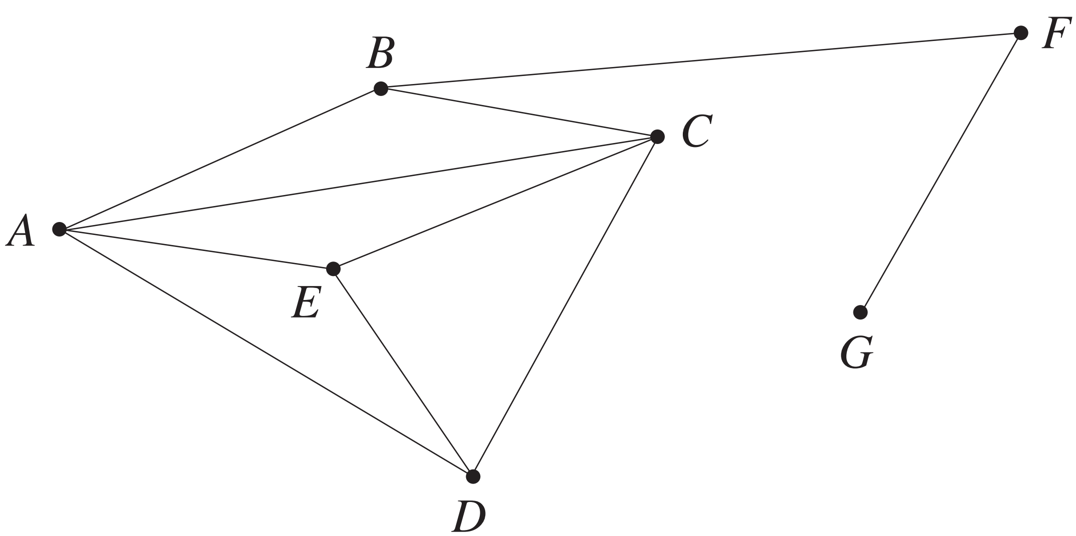
Some problems can be solved by coloring the vertices of an appropriate graph.

- **a** What do the vertices, edges, and colors represent in the graph that you used to solve the radio-station problem?
- **b** How did "coloring a graph" help solve the radio-station problem?
- **c** Describe some strategies or algorithms that you used to color the graph.
- **d** In what ways can two graphs differ and yet still both accurately represent a situation?

***Be prepared to share your thinking and coloring algorithm with the class.***

# Check for Understanding



## Activity 5
## Debrief and Review ESC
Time: 30 minutes

**Overview and Rationale**
This activity serves to debrief the previous problem-based instructional task and also review the fundamental components of Every Student Counts. In particular, this activity serves to launch the new ESC formative assessment component, which is called Assessment for Learning.

**Connections**
This activity uses the previous activity as a springboard to review the fundamental components of ESC. These components will be revisited in all subsequent activities.

**Conducting the Activity**
Present and briefly discuss the slides for Debrief and Review ESC in the Powerpoint file: Day1slides.ppt. In particular:
- Small groups should briefly identify and discuss the characteristics of a problem-based instructional task as seen in the previous activity on vertex-edge graphs. (Note: Related to "important mathematics" in the first characteristic, vertex-edge graphs are undoubtedly important mathematics from the standpoint of contemporary applications and modern mathematics. In addition, the importance of this topic needs to be discussed in terms of the reality of current curricula and tests, which can make it a challenge to implement vertex-edge graphs in the high school mathematics curriculum.)
- Review and discuss meaningful distributed practice and the ESC lesson format, using examples from the previous activity.
- Discuss the new ESC assessment component – Assessment for Learning. Present and discuss the slides about research, focus, and ESC implementation.

**Materials**
- Slides in the Debrief and Review section of the Powerpoint file: h_y2-d1_slides.ppt.

## Activity 6
## Problem-Based Instructional Task 2
## Visualize, Construct, and Represent 3D Shapes
## Build, Visualize, and Draw Cube Structures

Time: 55 minutes

**Overview and Rationale**

This activity is the 2nd problem-based instructional task for today. It illustrates the theme of visualizing, constructing, and representing three-dimensional shapes.

**Connections**

This activity will illustrate, reinforce, and apply many of the points in the previous activity, Debrief and Review ESC. It also addresses the 2nd main mathematical theme for today about geometry.

**Conducting the Activity**

Participants should work in small collaborative groups to complete the lesson on Visualizing and Sketching Three-Dimensional Shapes. (See Teaching Master TM-12.) This should be carried out as a classroom simulation, with the workshop instructor acting as the teacher and the participants engaged as students. In particular:

- The lesson is organized in the ESC lesson format: Launch-Explore-Summarize-Check For Understanding. Be sure to model all phases of this lesson format.
- As you circulate, be sure to ask good questions and reinforce good group behavior.
- As the mathematics is being investigated, be sure to press for good answers to problems 1c, 2d, and 3d.
- Follow-up discussion on the mathematics in the lesson: Briefly present and discuss the slide on Multiple Methods for Visualizing and Drawing 3D. This gives a broad overview and connects to related activities at the elementary and middle school levels of ESC.
- Point out the focus questions at the beginning of the lesson, and discuss how they are reflected in the Summarize and the Check for Understanding. Relate this to the Check for Understanding section on the ESC lesson plan template in which the teacher should consider: What to assess, How to assess, Results, and What to do with the resulting information in terms of adjusting instruction.
- Follow-up discussion on assessment: Point out the role and examples of assessment in this activity – Launch questions (e.g., help you assess what students know coming into the lesson), questioning during the Explore phase, Summarize questions, and the Check for Understanding tasks. Emphasize that the focus on assessment in Every Student Counts is assessment for learning.

**Materials**

- TM-12: Visualizing and Sketching Three-Dimensional Shapes
- One slide from the Powerpoint file: Day1slides.ppt
- Models of boxes and the “double square pyramid” shape from the CFU tasks

# Visualizing and Sketching Three-Dimensional Shapes

## Launch

It is not always practical to construct models of three-dimensional shapes. For example, you cannot fax a scale model of an off-shore oil rig to an engineer in another country. Rather, the three-dimensional shape needs to be represented in two dimensions in a way that conveys the important information about the shape.

a What other examples can you think of where a two-dimensional drawing is more practical than a model of the three-dimensional shape?

b How can a three-dimensional shape be drawn in two dimensions?

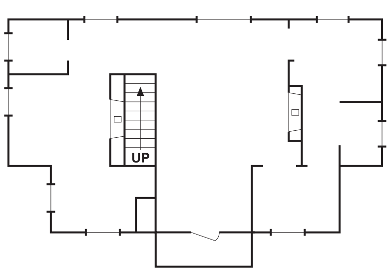
c What types of information about the three-dimensional shape can be conveyed in the two-dimensional drawing? What kind of information cannot be conveyed?

## Explore

In this investigation, you will explore two main questions:

*What are some effective ways to sketch three-dimensional shapes?*
*What information does each kind of sketch provide about the shape?*

There are several methods for representing a three-dimensional shape in a sketch, but since the sketch has only two dimensions some information about the three-dimensional shape will necessarily be missing. One way to depict three-dimensional shapes is to sketch two-dimensional *face-views* such as a top view, a front view, and a right-side view. Architects commonly use this method, called an **orthographic drawing**. For the house at the right, a *top view*, a *front view*, and a *right-side view* are shown below. Together, these views display the length, depth, and height of the building to scale. (You'll notice the top view is different from the other two. Floor plans such as this are frequently used instead of an exterior top view.)

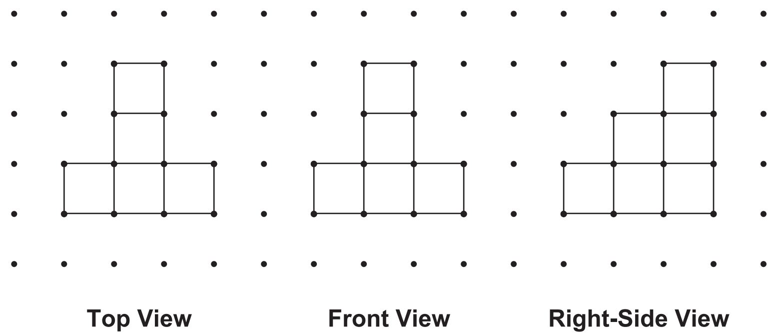



Top View

Front View

Right-Side View

**1.** Below is an orthographic drawing of a model of a hotel made from cubes.

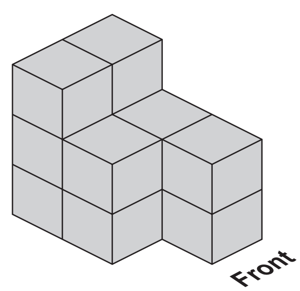



**a.** How many cubes make up the model?

**b.** Use blocks or sugar cubes to make a model of this hotel. Build your model on a sheet of paper or poster board that can be rotated.

**c.** Could you make the model using information from only two of these views? Explain.

**2.** Examine this model of a building built from cubes. Assume any cube above the bottom layer rests on another cube and that there are no hidden cubes.



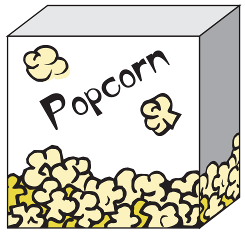

**a.** Use blocks to construct a physical model of this building.

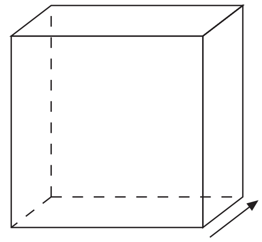
**b.** Make an orthographic sketch of this model.

**c.** How many cubes are in this model?

**d.** Would it be possible to make a model with fewer cubes that has the same top, front, and right-side views as this one? If so, explain how.

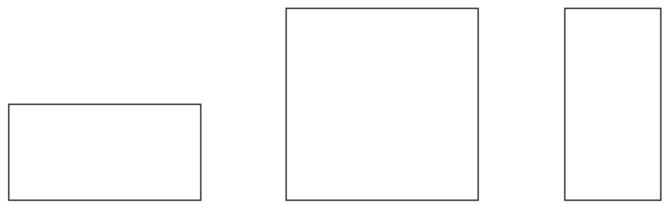
**3.** Another way to represent a three-dimensional shape such as a popcorn box is shown below. The sketch on the right, called an **oblique drawing**, is a *top-front-right corner* view of the box as a geometer would draw it. The front face was translated in the direction of the arrow shown to produce the back face, then edges were drawn to connect vertices. The sketch gives a sense of depth even though it is not drawn in true perspective. **Hidden lines**, such as the three edges of the box blocked from view, are shown as *dashed lines*.



**a.** What three-dimensional shape is the popcorn box?

**b.** Scale drawings of three face-views of the box are given below. If the actual box is 10 inches high, find its actual length and width by making appropriate measurements in the given face-views.



**Top View** **Front View** **Right-Side View**

**c.** Now examine more carefully the sketch above of the box from a top-front-right corner view.

- **i.** What appear to be the shapes of the faces as shown in the drawing? What are the shapes of the faces in the real box?
- **ii.** What edges are parallel in the real box? Are the corresponding edges in the sketch drawn parallel?

**d.** Sketch the box from a bottom-front-left corner view. *Hint:* Change the dashed edges in the above drawing to solid, and make three of the solid edges dashed.

## Summarize

A three-dimensional shape can be represented in two dimensions in various ways.

**a** Describe two ways to represent a three-dimensional shape in two dimensions.

**b** Discuss the similarities and differences between an oblique drawing of a box and the box itself.

**c** What information about the shape do you get from an orthographic drawing that you don't get from an oblique drawing? Vice versa?

***Be prepared to share your ideas with the class.***

## Check for Understanding

1. Build a model using 6 cubes.
   a. Make an orthographic drawing of this model.
   b. Make an oblique drawing of this model.

2. Consider a three-dimensional shape that is made up of two square pyramids sharing a common base. Make an orthographic drawing of this shape. Assume an edge of the common base is parallel to an edge of your desk.

# Activity 7
## Problem-Based Instructional Task 3
## Use Visualization and Reasoning to Solve Problems
## How Many Regular Polyhedra (Platonic Solids) Are There?

Time: 55 minutes

**Overview and Rationale**

This activity is the 3rd and last problem-based instructional task for today. It illustrates the theme of using visualization and reasoning to solve problems.

**Connections**

This activity will illustrate, reinforce, and apply many of the points in the previous activity, Debrief and Review ESC. It also addresses the 3rd main mathematical theme for today about geometry.

**Conducting the Activity**

Participants should work in small collaborative groups to complete the lesson on How Many Regular Polyhedra (Platonic Solids) Are There?. (See Teaching Master TM-13.) This should be carried out as a classroom simulation, with the workshop instructor acting as the teacher and the participants engaged as students. In particular:

- The lesson is organized in the ESC lesson format: Launch-Explore-Summarize-Check For Understanding. Be sure to model all phases of this lesson format.
- As you circulate, be sure to ask good questions and reinforce good group behavior.
- In problem 2, you may provide rulers and a regular polygon template (TM-14).
- Make sure participants have good explanations for problem 3.
- You will need multiple copies of the cutout templates for the regular polyhedra (TM 15-19). Do not hand out these templates until participants have worked through the mathematics and are ready for them – see problem 4.
- For the Check for Understanding tasks involving applets, have students demonstrate each applet and explain what is shown.
- Follow-up discussion on assessment, as time permits: Point out the role and examples of assessment in this activity – Launch questions (e.g., help you assess what students know coming into the lesson), questioning during the Explore phase, Summarize questions, and the Check for Understanding tasks. Also, point out the focus question at the beginning of the lesson, and discuss how it is reflected in the Summarize and the Check for Understanding sections. Emphasize that the focus on assessment in Every Student Counts is *assessment for learning*.

**Materials**

- TM-13: How Many Regular Polyhedra (Platonic Solids) Are There?
- TM 14: Regular polygon template
- TM 15-19: Cutout templates (nets) for the regular polyhedra
- Models of the regular polyhedra (to show at the end of the lesson)
- Scissors, tape, glue sticks, rulers
- Access to the Internet applets in the CFU tasks

# How Many Regular Polyhedra (Platonic Solids) Are There?

Exercising your reasoning and visualization skills
to solve a famous problem in geometry

## Launch

There are many interesting and useful geometric shapes. Some shapes seem to be "regular" while others are "irregular." Think about "regular" 2D and 3D shapes.

**a** Show or describe some 2D or 3D shapes that you think are "regular." What makes them "regular?"

**b** Shapes like you described in Part a are important because, for example, they are often used as the building blocks of other shapes. So we define them and study their properties. Thus, a **regular polygon** is a very specific type of polygon. Perhaps you already know about regular polygons. If so … Give an example of a regular polygon. What is the definition of a regular polygon? How many regular polygons are there?

**c** Roughly, a regular polyhedron is a 3D shape made up of regular polygons. How many regular polyhedra do you think there are?

## Explore

The three-dimensional counterpart of a regular polygon is a regular polyhedron. A **regular polyhedron**, also called a **Platonic solid**, is a convex polyhedron in which all faces are congruent, regular polygons. Furthermore, the arrangement of faces and edges is the same at each vertex (corner). The ancient Greeks speculated that the Platonic solids, which are named for the Greek philosopher Plato, are the shapes of the fundamental components of the universe.

Recall that there are infinitely many different regular polygons, named by the number of sides—regular (or equilateral) triangle, regular quadrilateral (or square), regular pentagon, regular hexagon, and so on. How many regular polyhedra are there? This is one of the most famous problems in geometry. In this investigation, you will solve this problem:

*How many differently-shaped regular polyhedra are possible and why?*

**1.** A good way to build and count polyhedra is to think about corners, and the faces that meet at a corner.

**a.** Think about any corner of any polyhedron. Several faces meet at a corner. What is the *fewest* number of faces that must meet at a corner of a polyhedron? Explain.

**b.** Think about creating a 2D drawing of a corner and the faces that meet at the corner. That is, think about choosing one corner and the attached faces and flattening the attached faces on a table top, creating a gap as needed. Or, another way to think about this is to draw the corner and faces on a sheet of paper as a diagram that could be folded up to make the corner. Such a diagram is called a **net**.

- Sketch a drawing like this for one corner of a cube.
- Cut out and fold the net to form one corner of a cube.

**c.** In Part b, you sketched a net for one corner of a cube. What is the sum of the measures of the angles of the squares that meet at the one corner?

**d.** Think about creating a net for any corner of a polyhedron, and then measuring the sum of the face angles around that corner. What is the *largest* such sum possible, for one corner of any polyhedron? Explain.

**2.** Review the information you discovered in Part a and Part d of Problem 1. Make sure everyone in your group agrees and can explain the answers to those questions. Now you will use this information to construct corner nets for all possible regular polyhedra.

**a.** The faces of a regular polyhedron must all be congruent regular polygons. For example, all the faces could be triangles. The same number of triangles must meet at each corner.

- What is the fewest number of triangles that could meet at a corner? Draw a net for such a corner.
- Consider a polyhedron in which there is one more triangle meeting at corner. Draw a net for such a corner.
- Continue adding one more triangle at a corner and drawing the corresponding net, for as many triangle faces as possible for one corner.

**b.** Now consider a regular polyhedron where all faces are squares. Draw a corner net for each possible polyhedron like this.

**c.** Continue with this strategy of drawing corner nets, using all possible regular polygons.

**3.** Examine all the different corner nets you drew in Problem 2. Use this work to answer the main question: How many regular polyhedra are there? Justify your answer.

**4.** When your group is satisfied with your answer and justification in Problem 3, it's time to construct the regular polyhedra. Send one member of your group to pick up the Platonic Solids nets. The rest of your group should gather together the other material and equipment you will need – at least one pair of scissors, one roll of tape, and one glue stick. Build the following polyhedra:

- Each member of your group should build ONE OF THE FOLLOWING: tetrahedron, cube (also called hexahedron), octahedron.
- If you finish earlier than others, build a dodecahedron or an icosahedron.

## Summarize

In this investigation, you investigated regular polyhedra, also known as Platonic solids.

(a) What is a regular polyhedron? How many regular polyhedra are there? What are their names?

(b) For each regular polyhedron, describe a face and give the number of faces that meet at each vertex (corner).

(c) Explain why there cannot be more than five differently-shaped regular polyhedra.

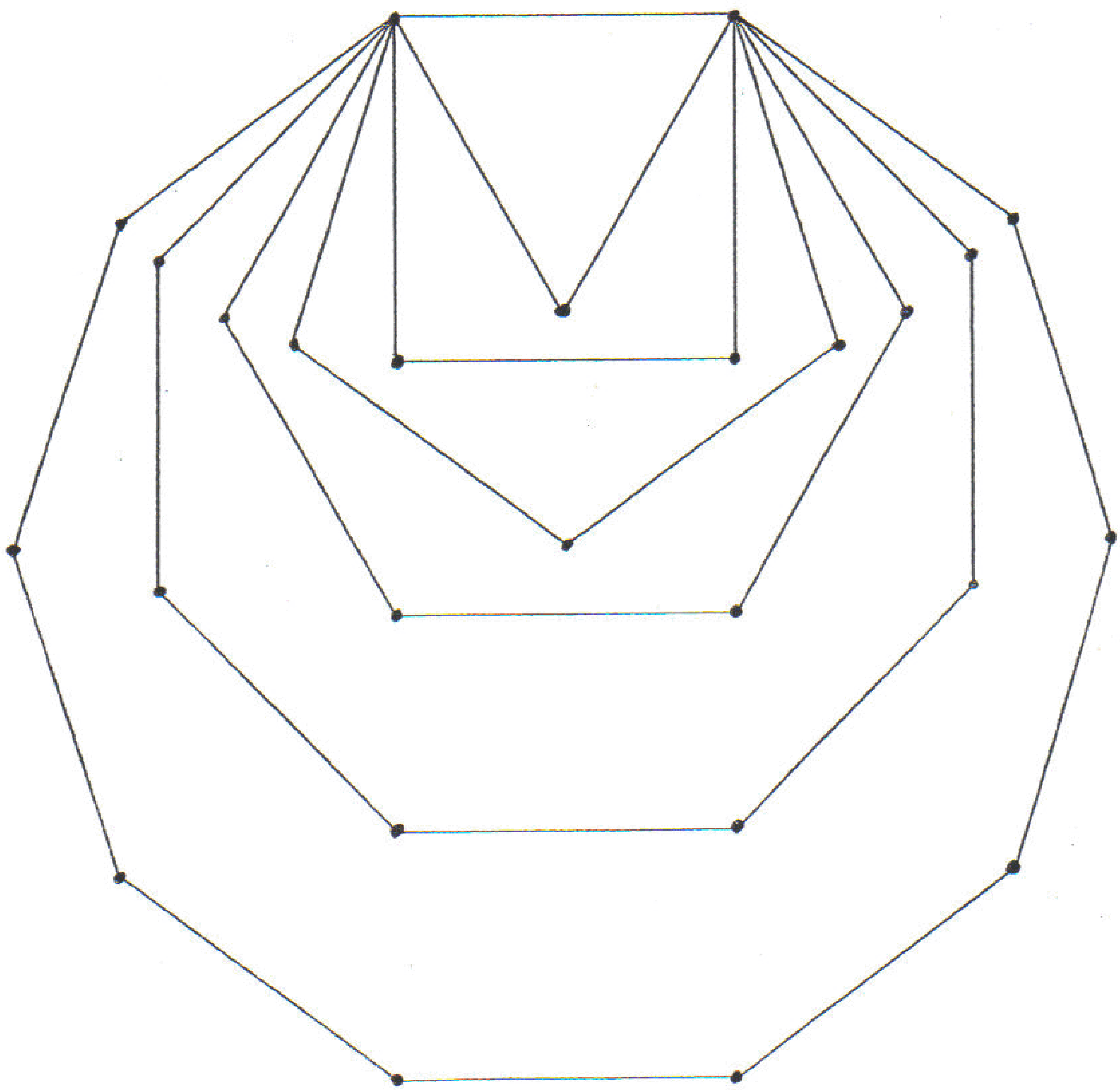
***Be prepared to share your answers with the entire class.***

## Check for Understanding

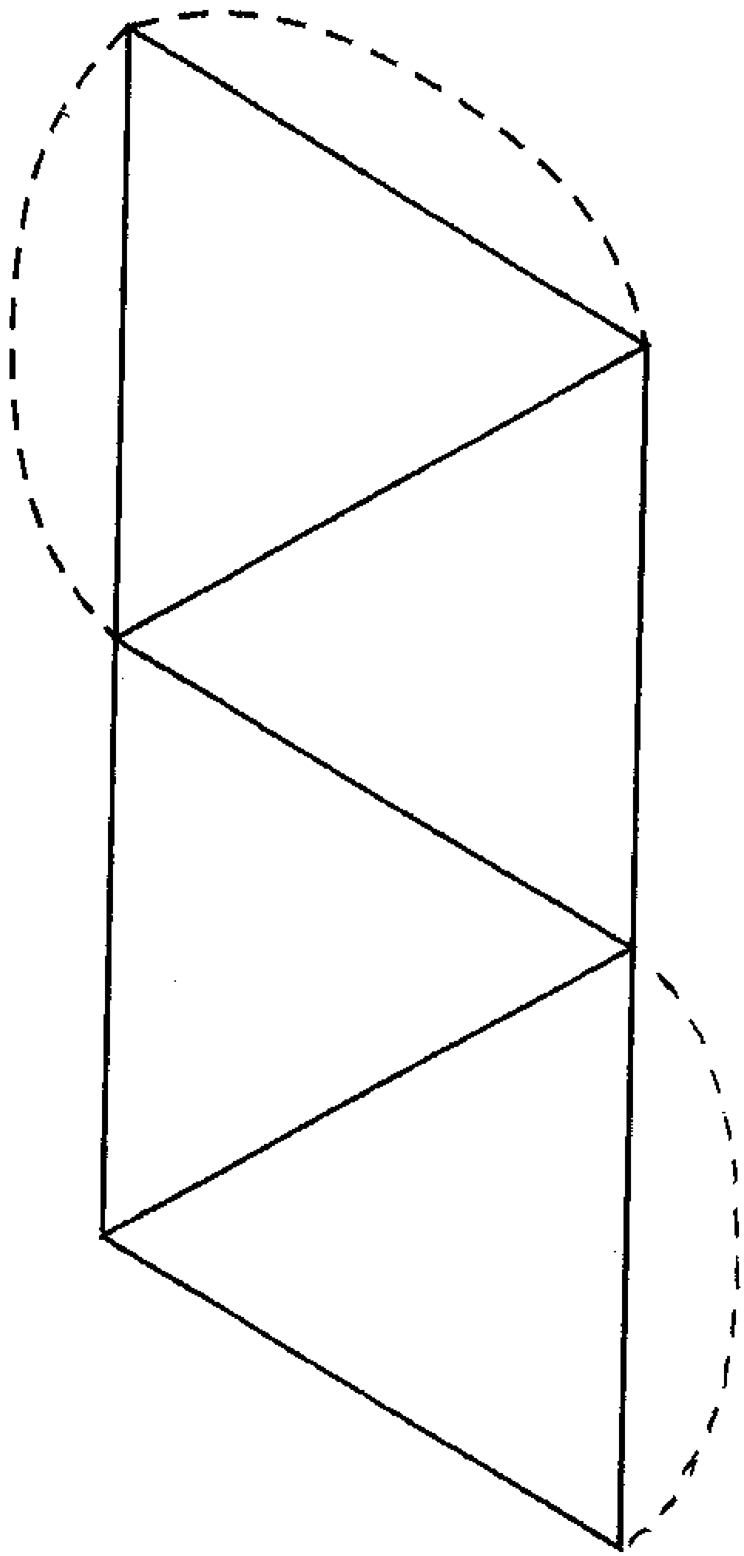
**a.** Check out the "Tiling" applet at:
http://www.wmich.edu/cpmp/applets/tiling2.html
Use the applet to demonstrate and explain the answer to the question: How many regular polyhedra are there?

**b.** Check out the "Geometric Solids" applet at:
http://illuminations.nctm.org/tools/tool_detail.aspx?id=70
Use the applet to show all the regular polyhedra.

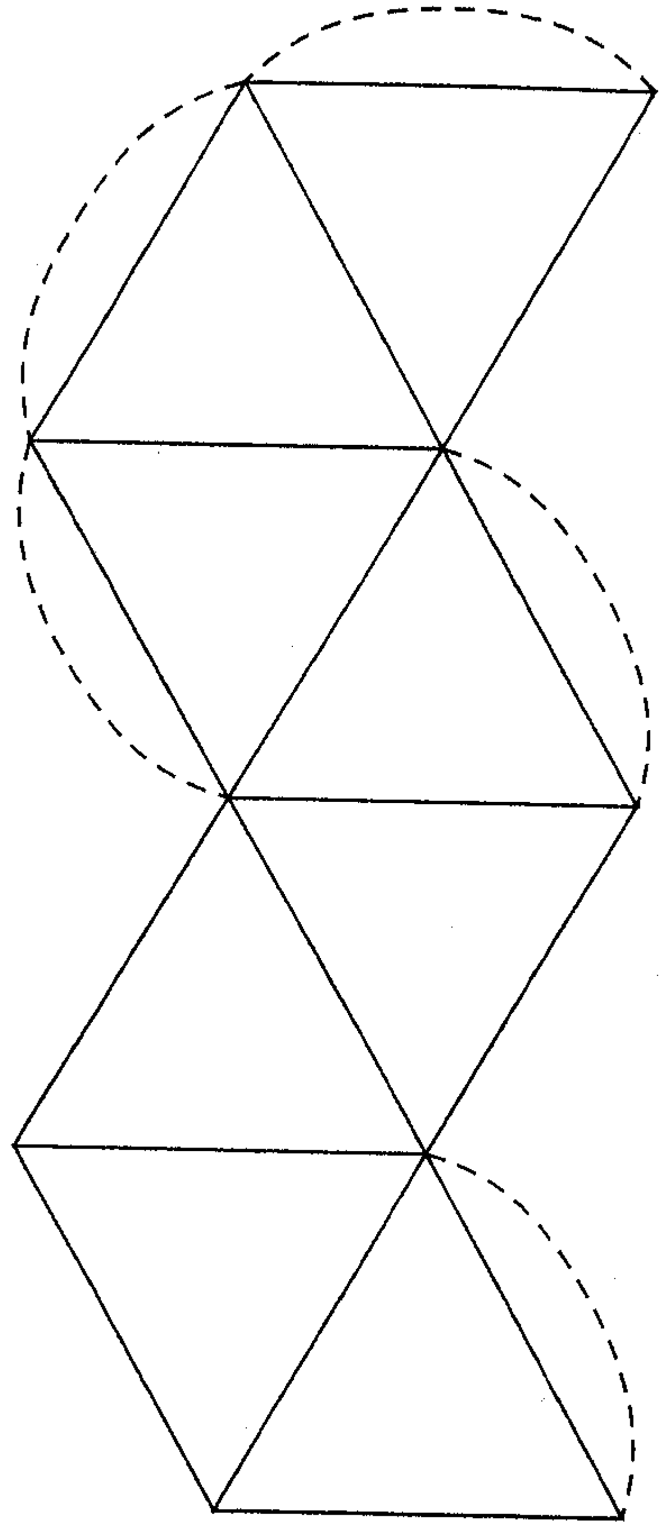
# Regular Polygon Template



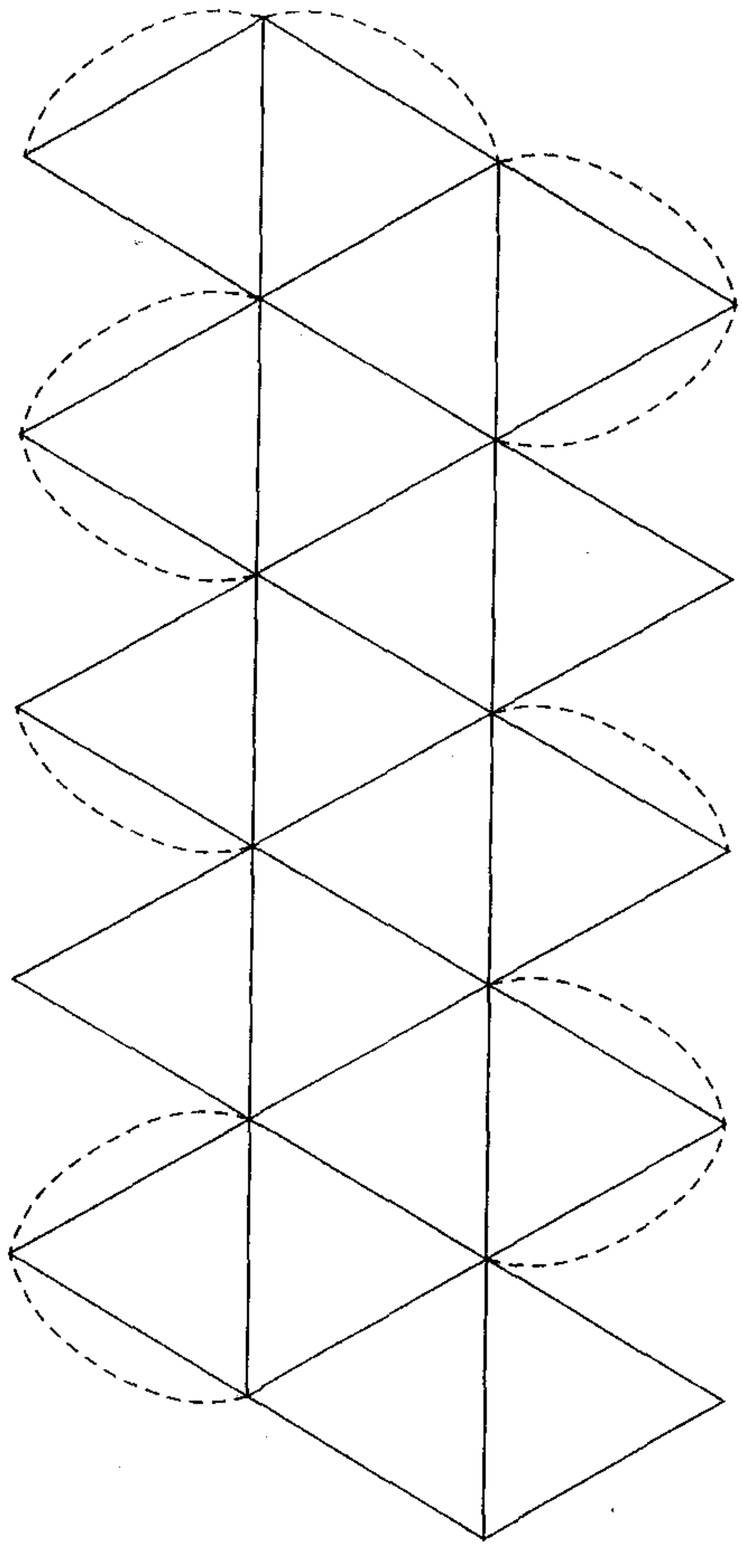
# Tetrahedron



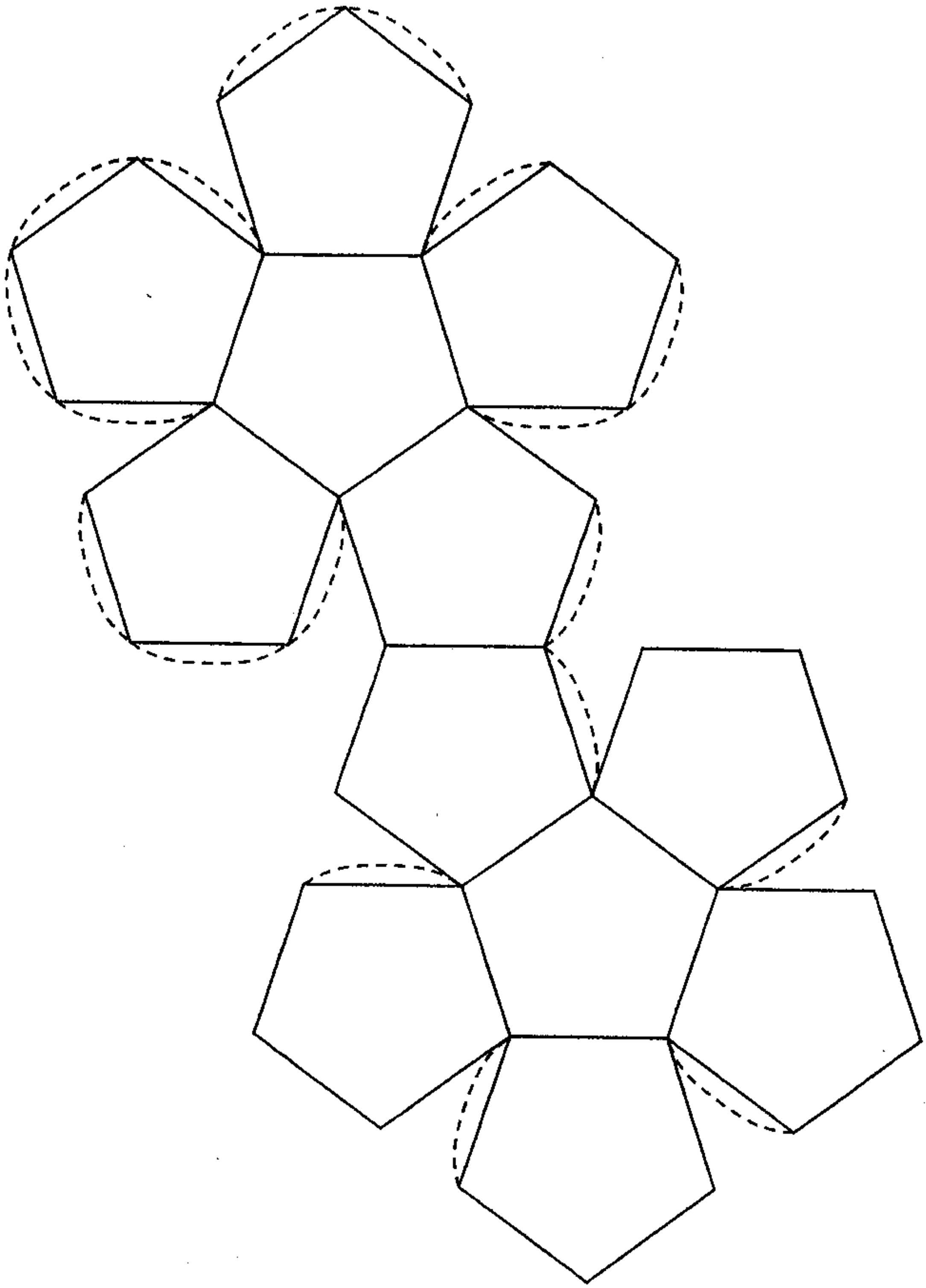
# Octahedron



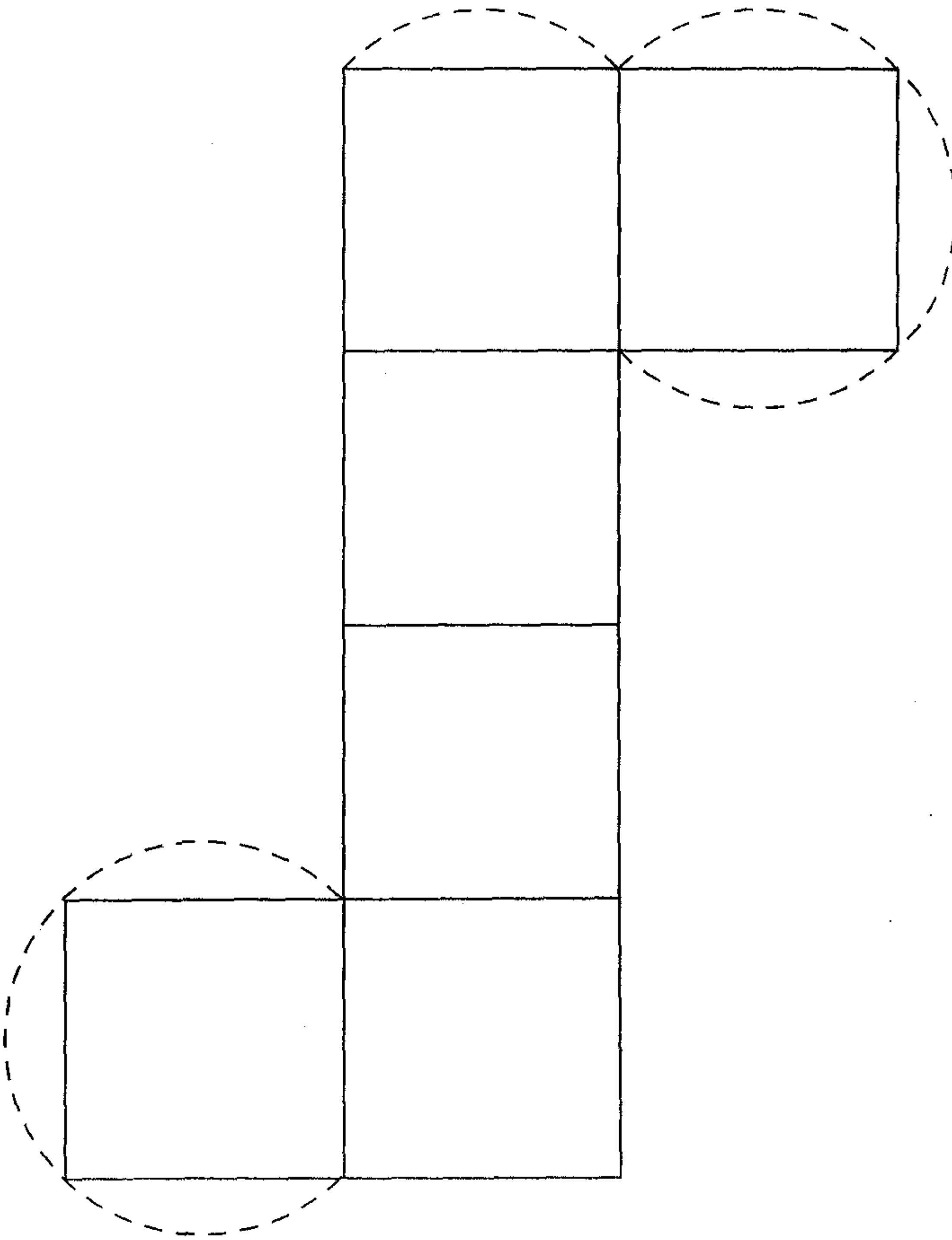
# Icosahedron



# Dodecahedron



# Hexahedron



# Activity 8
# Closure, Assignments, Evaluations

Time: 15 minutes

**Overview and Rationale**
It is important to end the day with some time for summary and closure. This is accomplished by briefly presenting and reviewing the NCTM geometry standard for the day, the NCTM principle, and the NCTM process standard. Also, make assignments and administer workshop evaluations.

**Connections**
This activity provides a broad summary of all activities and brings closure to whole day.

**Conducting the Activity**
Present and briefly discuss the final slides in the Powerpoint file: Day1slides.ppt. In particular:

- Present and review the NCTM geometry standard for the day: Use visualization, spatial reasoning, and geometric modeling to solve problems. Briefly connect to the activities of the day.
- Present and review the NCTM principle for the day: Equity. Briefly connect to the activities of the day.
- Present and review the NCTM process standard for the day: Representation. Briefly connect to the activities of the day.

Also, give the assignment for Day 2, and administer the workshop evaluation forms for today's workshop.

**Materials**

- Final slides in the Powerpoint fileh_y2-d1_slides.ppt.
- Agenda with assignment printed at the bottom
- TM-20: Reading Assignment for Day 2
- Workshop evaluation forms

# Reading Assignment for Day 2

**Assessment**

1. Take the short assessment entitled "Some Problems about Graphs," handed out on Day 1.
   - This assessment is discussed in the next reading. Be sure to take the assessment yourself before reading the article in #2 below.
   - As part of the first Classroom Practice assignment, administer this assessment to a class of high school algebra students. (See details in the Classroom Practice assignment description on the bottom of the Day 1 Agenda page.)
2. Read "Examining Students' Reluctance to Use Graphs" by Van Dyke and White in *Mathematics Teacher,* Vol. 98, No. 2, September 2004, pp. 110-117.
   - This article looks at incoming calculus students' understanding of Cartesian coordinates and the graphical representations of functions.
   - As you read this article, compare to the results when you took the assessment and when high school algebra students took the assessment. (See #1 above.)
3. Read "Section 2: Using Effective Questioning Techniques" by the Qualifications and Curriculum Authority (QCA) in *Assessment for Learning: Using Assessment to Raise Achievement in Mathematics at key stages 1-3* (QCA, 2003), pp. 8-14. Available as a PDF download at: http://www.qca.org.uk/293_3233.html
   - This publication aims to provide guidance on implementing assessment for learning, based on the research and previous publications by Black and Wiliams ("Black Box" articles).
   - Compare the questioning techniques presented in this article to the questioning you observe in a high school geometry class. (See details in the Classroom Practice assignment description on the bottom of the Day 1 Agenda page.)

**NCTM Principle for Day 2**

4. Read the section on the Technology Principle in Chapter 2: Principles for School Mathematics from *Principles and Standards for School Mathematics*, pp. 24-27.
   - The Technology Principle states that: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning."
   - Reflection Question: There are three main points made in the description of the Technology Principle. What is an example of each point that you have seen in mathematics classrooms?

**NCTM Process Standard for Day 2**

5. Read "Connections" for Grades Pre-K–12 (pp. 64-66) and for Grades 9–12 (pp. 354-359) in *Principles and Standards for School Mathematics*.
   - The Connections Standard consists of three main goals. Be prepared to analyze the Day 2 activities in terms of the three goals of the Connections Standard.
   - Reflection Question: According to these readings, what is the teacher's role in developing connections in mathematics classes?

**NCTM Content Standard from Day 1**

6. Read the section entitled, "Use visualization, spatial reasoning, and geometric modeling to solve problems" in the Geometry section for Grades Pre-K–12 (p. 43) and for Grades 9-12 (pp. 315-318) in *Principles and Standards for School Mathematics*.
   - We will read the content standard that we focused on in Day 1, so that we can relate the standard to geometry experiences from Day 1.
   - Reflection Question: What are some activities from Day 1 that provide examples of points made in this reading? What additional points were made in Day 1 that complement or extend the points made in this reading?